


Joe's Color Convention

Black

- ↳ Generic
- ↳ Question
- ↳ Title

Yellow

- ↳ temporary note

Blue

- ↳ Sub title
- ↳ Graph axis
- ↳ Time domain

Light Blue

- ↳ Sub color for Blue
- ↳ Used for graphs/paragraphs

Red

- ↳ Sub title
- ↳ Counterpart of Blue
- ↳ Frequency Domain

Pink

- ↳ Sub Color for Red
- ↳ Used for graphs/paragraphs

Green

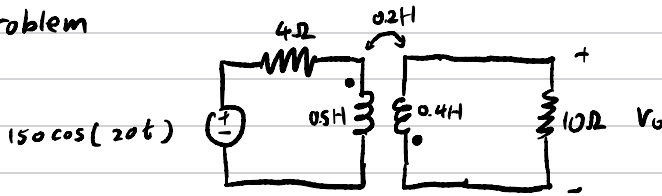
- ↳ Sub title
- ↳ Used for examples

Lime

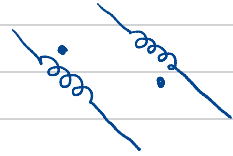
- ↳ Sub color for Green

Magnetic Coupled Circuits

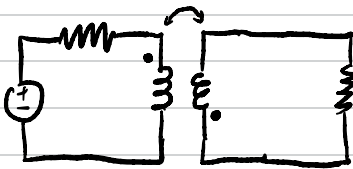
Problem



Activity :



Find V_o in time domain.



Find coupling coefficient k & instantaneous energy stored at $t = 2$

instantaneous energy stored:

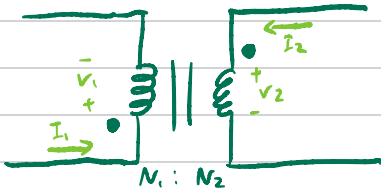
$$w = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2$$

Coupling coefficient

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

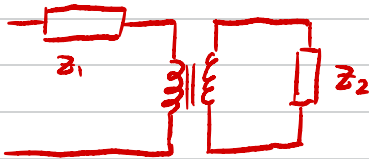
Magnetic Coupled Circuits

Ideal transformers



$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

Load reflecting

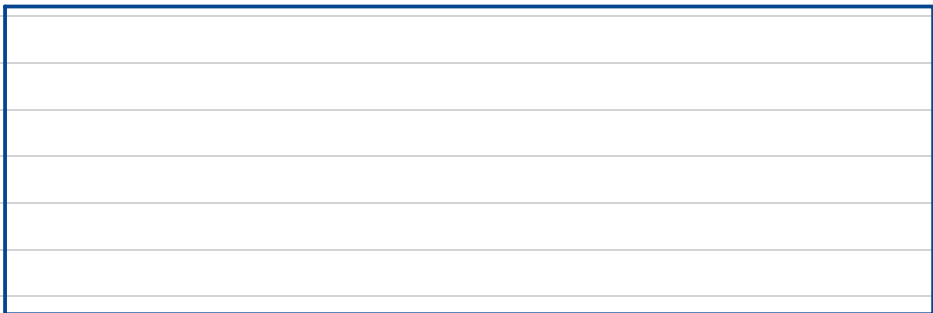
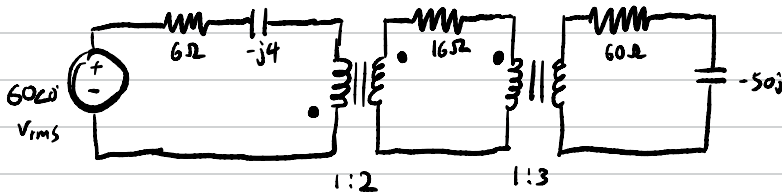


Derive an equation for reflected Load



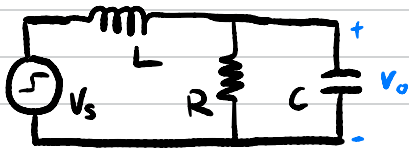
Problem

Find the complex power supplied by the source in the circuit below



RLC step response

Problem

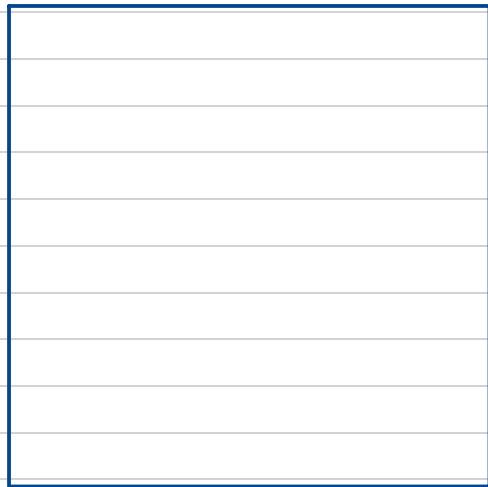
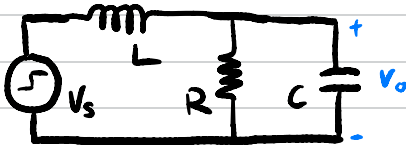


Consider the above circuit. With $R = 1000\Omega$, $L = 10\text{ mH}$ & $C = 100\text{ nF}$. With $V_s = 5\text{ u}(t)\text{ V}$

Find an expression for V_o in terms of t .

4 Procedure

Find the second order differential equation

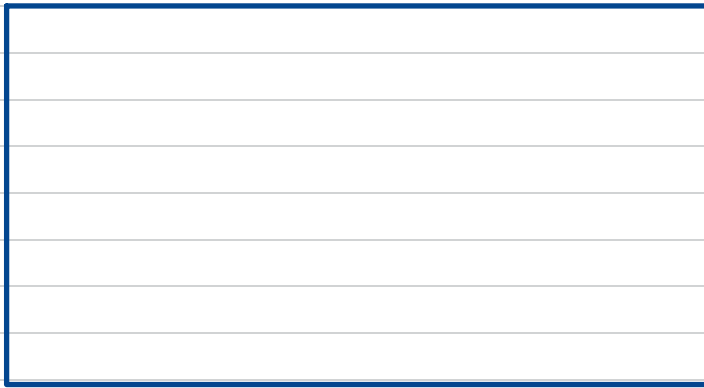


RLC step response

Find α , ω_n & ω_d

Note a 2nd ODE can be expressed as:

$$\frac{d^2x}{dt^2} + 2\alpha \frac{dx}{dt} + \omega_n^2 x = f(t)$$

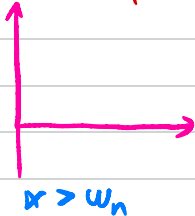


$$R = 1 \text{ k}\Omega$$

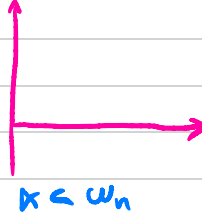
$$L = 10 \text{ mH}$$

$$C = 100 \text{ nF}$$

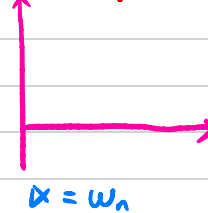
Overdamped



Underdamped



Critically damped



what happens at $\alpha = 0$?

RLC step response

Solve for natural response (Homogeneous) $x_n(t)$

Method 1

solve characteristic equation $s^2 + 2\alpha s + \omega_n^2 = 0$

& get roots s_1 & s_2 .

Solution: $x_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

Method 2

Case

underdamped

overdamped

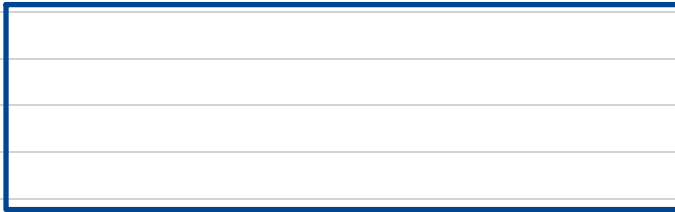
critically damped

Solution

$$x_n(t) = e^{-\alpha t} (B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t))$$

$$x_n(t) = A_1 e^{(s_1 + \alpha)t} + A_2 e^{(s_2 + \alpha)t}$$

$$x_n(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t}$$



Solve for forced response $x_f(t)$

Forced Response

$p(t)$ (deg n)

$p(t) e^{kt}$

$p(t) e^{kt} \cos(\ell t)$

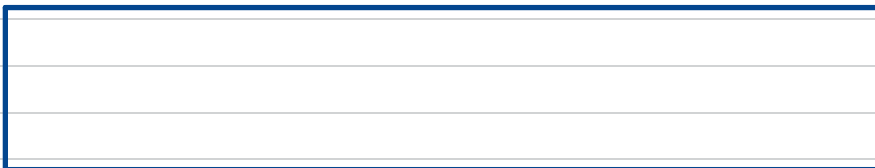
$p(t) e^{kt} \sin(\ell t)$

Guess for $x_f(t)$

$Q(t)$ (deg n)

$Q(t) e^{kt}$

$e^{kt} [Q_1(t) \cos(\ell t) + Q_2(t) \sin(\ell t)]$



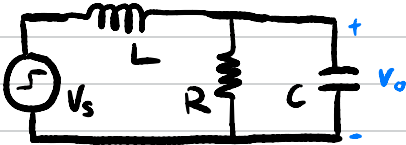
RLC step response

Complete the solution

$$x(t) = x_n(t) + x_f(t)$$

Find the constants

Use initial conditions to find unknowns



Laplace Transform

Essence of Laplace Transform



Laplace
Himself

Try Fourier Transforming
 $e^{xt} \sin \pi t$

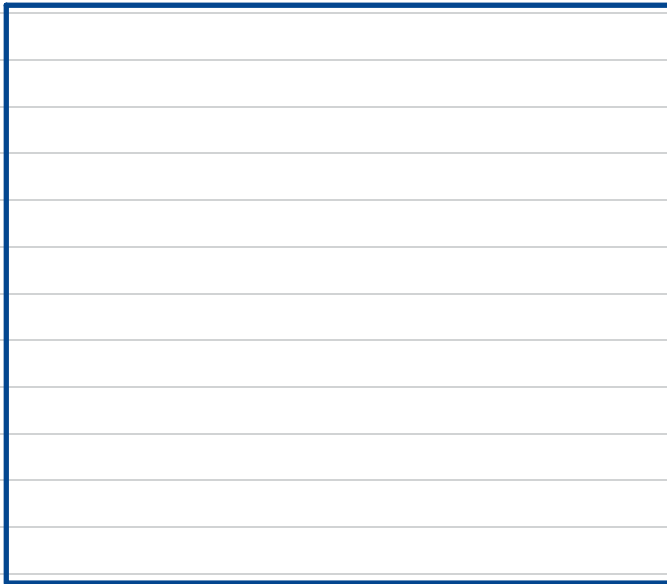
Laplace Transform

$$\mathcal{L}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

Inverse Laplace Transform

$$\mathcal{L}^{-1}[F(s)] = \int_{0-j\infty}^{0+j\infty} F(s) e^{st} ds$$

Using the definition of Laplace Transform
find the Laplace Transform of $f(t) = e^{-xt} u(t)$



Laplace Transform

Laplace Transform properties

Refer to formula sheet

use Laplace Transform properties to
make life easier!



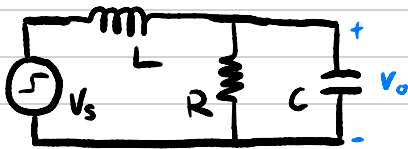
Find the Laplace Transform of $e^{-x_0} \sin(\omega t)$

Find the inverse Laplace Transform of

$$F(s) = \frac{(s+2)(s+4)}{s^2+3^2}$$

Laplace Transform

Problem



Consider the above circuit. With $R = 1000\Omega$,
 $L = 10\text{ mH}$ & $C = 100\text{ nF}$. With $V_s = 5u(t)\text{ V}$

Find an expression for V_o in the Laplace domain



Side note

$$\int e^{cx} \sin bx \, dx$$

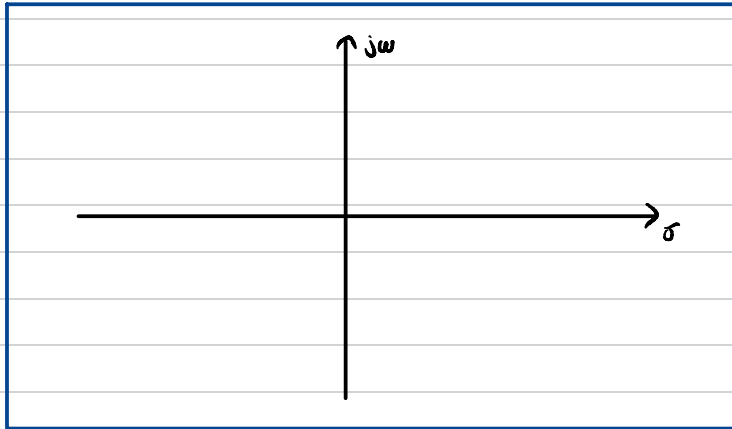
$$= \frac{e^{cx}}{c^2 + b^2} [c \cos bx + b \sin bx]$$

Laplace Transform

Problem

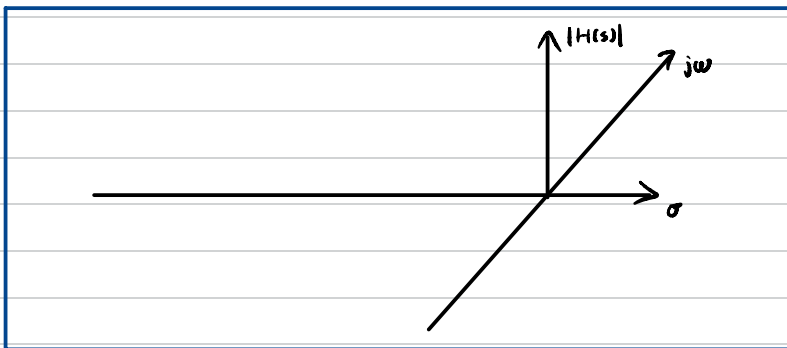
plot the following equation in the s -plane & roughly sketch the transfer function:

$$\frac{4(s+2)^2}{(s^2 + 6s + 25)}$$



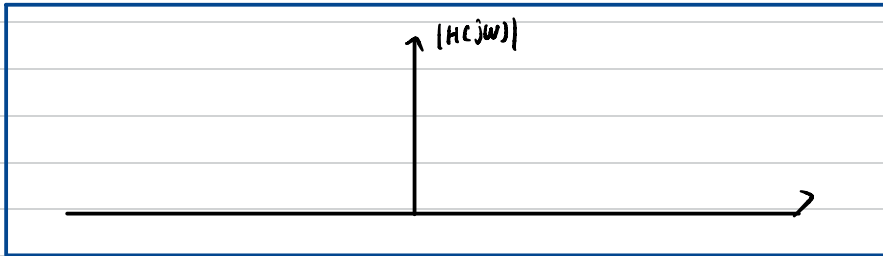
Drawing Activity!

draw a 3 dimensional s plane!



Laplace Transform

now roughly sketch the transfer function Amplitude spectrum

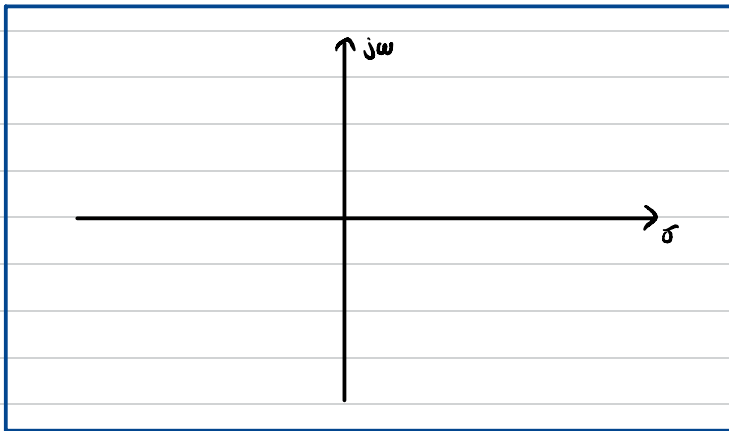


Problem

plot the following transfer function into the s plane.

Determine whether it is a stable system.

$$\frac{(s+3)^2(s^2+4s+13)}{(s-3)(s^2+3s+34)(s+6)}$$



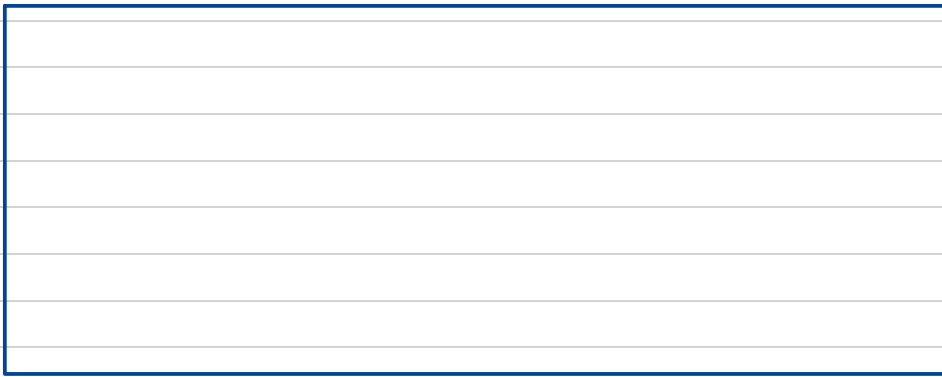
Two Port Network

Impedance Parameter

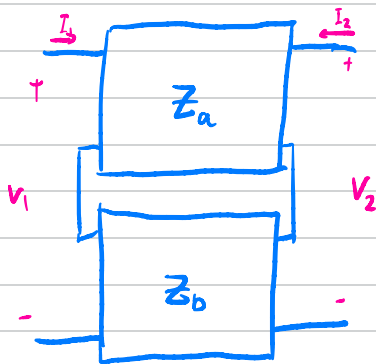


$$V = Z I$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



Application

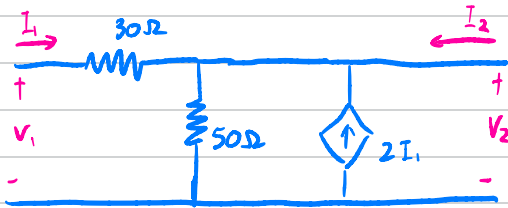


Consider the whole system find z parameters for system.

$$[z] = [z_a] + [z_b]$$

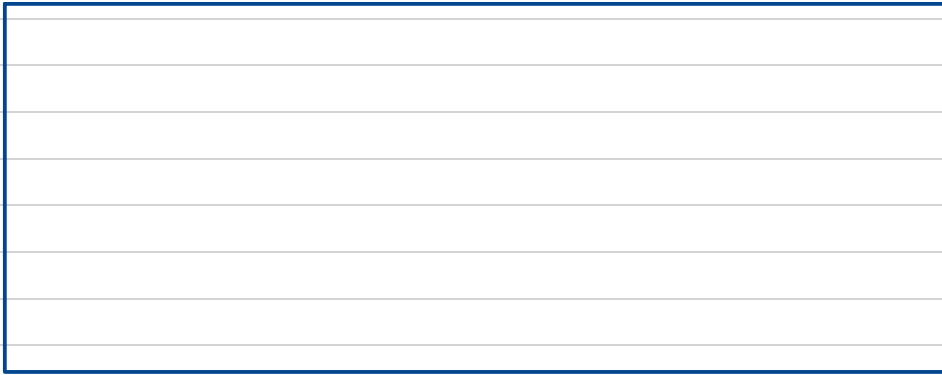
Two Port Network

Admittance Parameter

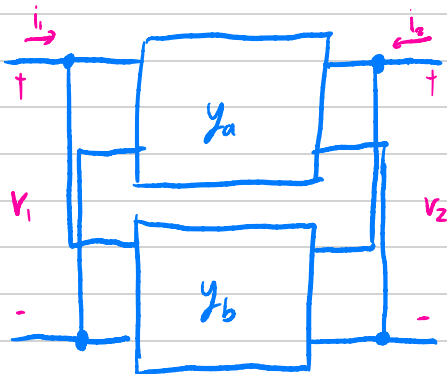


$$I = YV$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



Application

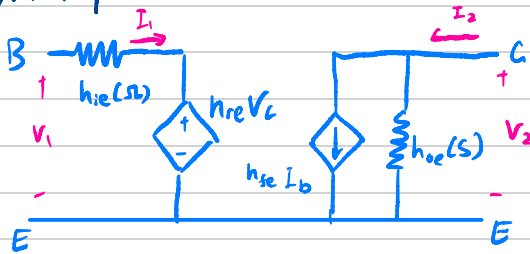


Consider the whole system find z parameters for system.

$$[y] = [y_a] + [y_b]$$

Two Port Network

Hybrid parameters



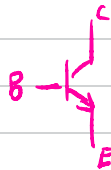
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

show $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{ie} & h_{re} \\ h_{se} & h_{oe} \end{bmatrix}$



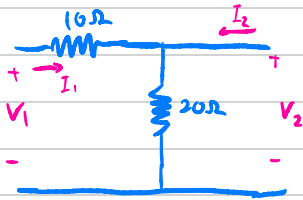
Application

h parameters tell us a lot of information about a transistor & its behaviour. This is later covered in ELEC 2133.

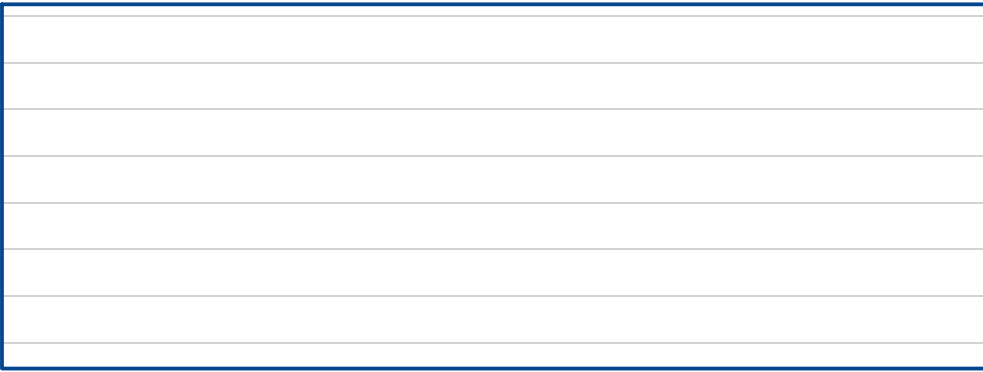


Two Port Network

Transmission Parameters



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



Application



$$[T] = [T_n][T_o]$$

problem

find the transmission parameter for the following circuit

