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# Joe's Color Convention

## Black

- ↳ Generic
- ↳ Question
- ↳ Title

## Yellow

- ↳ temporary note

## Blue

- ↳ Sub title
- ↳ Graph axis
- ↳ Time domain

## Light Blue

- ↳ Sub color for Blue
- ↳ Used for graphs/paragraphs

## Red

- ↳ Sub title
- ↳ Counterpart of Blue
- ↳ Frequency Domain

## Pink

- ↳ Sub Color for Red
- ↳ Used for graphs/paragraphs

## Green

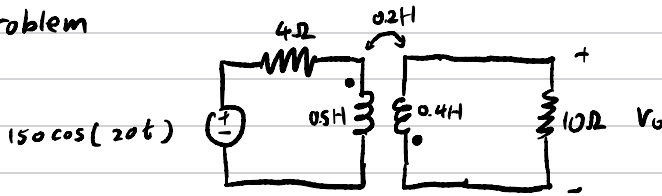
- ↳ Sub title
- ↳ Used for examples

## Lime

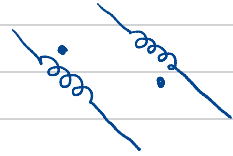
- ↳ Sub color for Green

# Magnetic Coupled Circuits

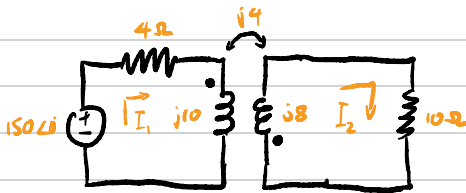
Problem



Activity:



Find  $V_o$  in time domain.



in  $I_1$

$$150 = 4I_1 + j10I_1 + j4I_2$$

$$150 = (4 + j10)I_1 + j4I_2$$

in  $I_2$

$$(j8 + 10)I_2 + (j4)I_1 = 0$$

$$I_1 = (2 - \frac{5}{2}j)I_2$$

$$150 = (4 + j10)(2 - \frac{5}{2}j)I_2 + j4I_2$$

$$= (33 + j14)I_2$$

$$I_2 = 4.18 \angle -0.401$$

$$V_2 = 41.8 \angle -0.401$$

$$v_2(t) = 41.8 \cos(20t - 0.401^\circ)$$

Find coupling coefficient  $k$  & instantaneous energy stored at  $t = 2$

instantaneous energy stored:

$$w = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2$$

Coupling coefficient

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$I_2 = 4.18 \angle -0.401 \rightarrow i_2(t) = 4.18 \cos(20t - 0.401)$$

$$I_1 = (2 - \frac{5}{2}j)I_2$$

$$= 13.40 \angle -1.30 \rightarrow i_1(t) = 13.40 \cos(20t - 1.30)$$

$$i_2(2) = -1.35$$

$$i_1(2) = 7.23$$

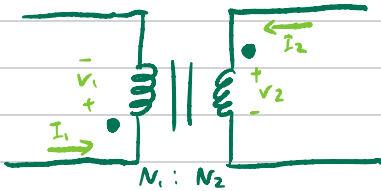
$$w = \frac{1}{2} 0.5 (7.23)^2 + \frac{1}{2} 0.4 (-1.35)^2 + 0.2 (-1.35)(7.23)$$

$$= 11.48 \text{ J}$$

$$k = \frac{0.2}{\sqrt{0.5 \cdot 0.4}} = 0.447$$

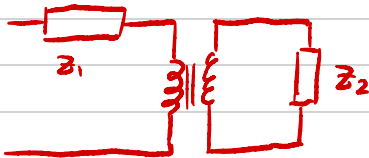
# Magnetic Coupled Circuits

## Ideal transformers



$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

## Load reflecting

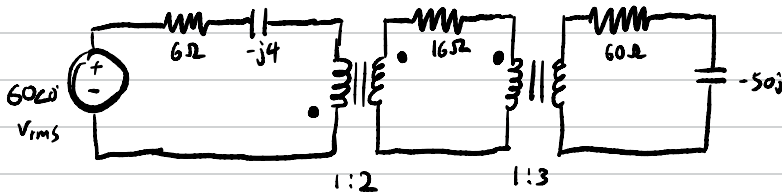


Derive an equation for reflected Load

$$\begin{aligned} Z_{RL} &= \frac{V_1}{I_1} & \text{for } 1:n \\ &= \frac{\left(\frac{N_1}{N_2}\right) V_2}{\left(\frac{N_2}{N_1}\right) I_2} & Z_{RL} = \frac{Z_2}{n^2} \\ &= \frac{Z_2}{\left(N_2/N_1\right)^2} \end{aligned}$$

## Problem

Find the Complex power supplied by the source in the circuit below



reflecting rightmost gives  $Z_{R1} = (60 - 50j) / 3^2$  New Load  
 $= \frac{20}{3} - \frac{50}{9}j$  → 22.67 - 5.57j

reflecting again gives  $Z_{R2} = (22.67 - 5.57j) / 2^2$   
 $= 5.67 - j1.39$

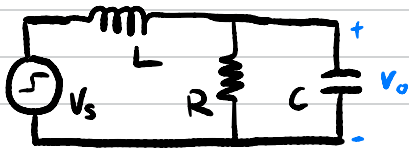
total impedance =  $6 - j4 + 5.67 - j1.39 = (11.67 - j5.39)$

$I = 60 / Z_{tot} = 4.24 + j1.96$ ,  $\therefore \text{Power} = VI^* = 254.3 - j117.5$



# RLC step response

## Problem

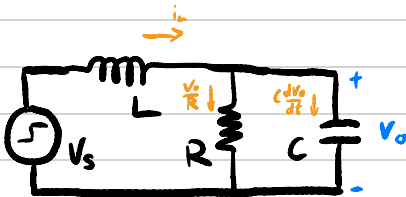


Consider the above circuit. With  $R = 1000\Omega$ ,  
 $L = 10\text{ mH}$  &  $C = 100\text{ nF}$ . With  $V_s = 5\text{ u}(t)\text{ V}$

Find an expression for  $V_o$  in terms of  $t$ .

## 4 Procedure

Find the second order differential equation



$$i_L = \frac{V_o}{R} + C \frac{dV_o}{dt}$$

$$\frac{di_L}{dt} = \frac{d}{dt} \left[ \frac{V_o}{R} + C \frac{dV_o}{dt} \right]$$

$$\frac{1}{L} L \frac{di_L}{dt} = C \frac{d^2 V_o}{dt^2} + \frac{1}{R} \frac{dV_o}{dt}$$

$$\frac{1}{L} [V_s - V_o] = C \frac{d^2 V_o}{dt^2} + \frac{1}{R} \frac{dV_o}{dt}$$

$$\frac{V_s}{LC} = \frac{d^2 V_o}{dt^2} + \frac{1}{RC} \frac{dV_o}{dt} + \frac{1}{LC} V_o$$

# RLC step response

Find  $\alpha$ ,  $\omega_n$  &  $\omega_d$

Note a 2<sup>nd</sup> ODE can be expressed as:

$$\frac{d^2 x}{dt^2} + 2\alpha \frac{dx}{dt} + \omega_n^2 x = f(t)$$

$\alpha$  - damping factor

$\omega_n$  - natural frequency

$\omega_d$  - damped frequency  $\omega_d = \sqrt{\omega_n^2 - \alpha^2}$

$$\frac{d^2 v_o}{dt^2} + \frac{1}{RC} \frac{dv_o}{dt} + \frac{1}{LC} v_o = \frac{V_s}{LC}$$

$$\alpha = \frac{1}{2RC} = 5000$$

$$\omega_n = \frac{1}{\sqrt{LC}} = 31623 \text{ rad/s}^{-1}$$

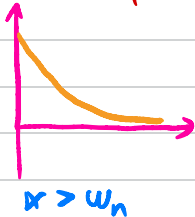
$$\omega_d = \sqrt{\omega_n^2 - \alpha^2} = 31225 \text{ rad/s}^{-1}$$

$$R = 1 \text{ k}\Omega$$

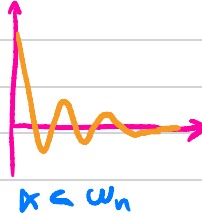
$$L = 10 \text{ mH}$$

$$C = 100 \text{ nF}$$

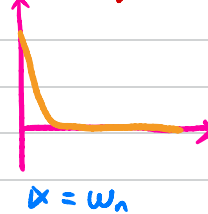
Overdamped



Underdamped



Critically damped



what happens at  $\alpha = 0$  ?

# RLC step response

Solve for natural response (Homogeneous)  $x_n(t)$

Method 1 (general)

solve characteristic equation  $s^2 + 2\alpha s + \omega_n^2 = 0$

& get roots  $s_1$  &  $s_2$ .

Solution:  $x_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

Method 2 (recommended)

Case

Solution

underdamped ( $\alpha < \omega_n$ )

$x_n(t) = e^{-\alpha t} (B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t))$

overdamped ( $\alpha > \omega_n$ )

$x_n(t) = A_1 e^{(\alpha + \omega_d)t} + A_2 e^{(\alpha - \omega_d)t}$

critically damped ( $\alpha = \omega_n$ )

$x_n(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t}$

Using Method 2

we know  $\omega_n = 31623 > \alpha = 5000$

response is underdamped

$\therefore x_n(t) = e^{-\alpha t} (B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t))$

Solve for forced response  $x_f(t)$

Forced Response

Guess for  $x_f(t)$

$p(t)$  (deg  $n$ )

$Q(t)$  (deg  $n$ )

$p(t) e^{kt}$

$Q(t) e^{kt}$

$p(t) e^{kt} \cos(\ell t)$

$e^{kt} [Q_1(t) \cos(\ell t) + Q_2(t) \sin(\ell t)]$

$p(t) e^{kt} \sin(\ell t)$

$V_s = 5 u(t)$  ( $V_s$  is a constant for  $t > 0$ )

Guess:  $V_f(t) = 5$   $\frac{d^2 V_0}{dt^2} + \frac{1}{RL} \frac{dV_0}{dt} + \frac{1}{LC} V_0 = \frac{5}{LC} = \frac{V_s}{LC}$

# RLC step response

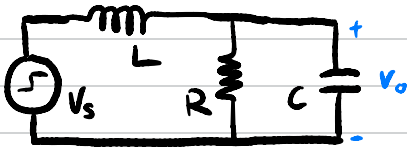
Complete the solution

$$x(t) = x_n(t) + x_f(t)$$

$$V_o(t) = e^{-\lambda t} (B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)) + 5$$

Find the constants

Use initial conditions to find unknowns



$$V_o(0^-) = V_s = 0$$

because of  $C$ ,  $V_o(0^+) = V_o(0^-) = 0$

$$V_o(0) = e^{-\lambda \cdot 0} (B_1 \cos(\omega_d \cdot 0) + B_2 \sin(\omega_d \cdot 0)) + 5$$

$$0 = B_2 + 5$$

$$B_2 = -5$$

$$V_o(t) = e^{-\lambda t} (-5 \cos(\omega_d t) + B_2 \sin(\omega_d t)) + 5$$

because of  $L$ ,  $i_L(0^-) = i_L(0^+) = 0$

$$i_C(t) = C \frac{dV_o}{dt}$$

$$= C [(-\lambda) e^{-\lambda t} [-5 \cos(\omega_d t) + B_2 \sin(\omega_d t)] + e^{-\lambda t} [-5 \sin(\omega_d t) + B_2 \omega_d \cos(\omega_d t)]]$$

substituting  $t=0$

$$0 = C [-\lambda(-5) + B_2 \omega_d]$$

$$B_2 = -0.8$$

$$V_o(t) = e^{-5000t} [-5 \cos(31225t) - 0.8 \sin(31225t)] + 5$$

# Laplace Transform

## Essence of Laplace Transform



Laplace  
Himself

Try Fourier Transforming  
 $e^{xt} \sin \pi t$

### Laplace Transform

$$\mathcal{L}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

### Inverse Laplace Transform

$$\mathcal{L}^{-1}[F(s)] = \int_{0-j\infty}^{0+j\infty} F(s) e^{st} ds$$

Using the definition of Laplace Transform  
find the Laplace Transform of  $f(t) = e^{-\alpha t} u(t)$

$$\begin{aligned}\mathcal{L}\{f(t)\} &= \int_{-\infty}^{\infty} e^{-\alpha t} u(t) e^{-st} dt \\&= \int_0^{\infty} e^{-(\alpha+s)t} dt \\&= \left[ \frac{e^{-(\alpha+s)t}}{-(\alpha+s)} \right]_0^{\infty} \\&= 0 - \frac{e^0}{-(\alpha+s)} \\&= \frac{1}{\alpha+s} \\F(s) &= \frac{1}{\alpha+s}\end{aligned}$$

# Laplace Transform

Laplace Transform properties

Refer to formula sheet

use Laplace Transform properties to make life easier!



Find the Laplace Transform of  $e^{-\alpha t} \sin(\omega t)$

$$\begin{aligned} \sin(\omega t) &\longleftrightarrow \frac{\omega}{s^2 + \omega^2} \\ e^{-\alpha t} f(t) &\longleftrightarrow F(s + \alpha) \\ e^{-\alpha t} \sin(\omega t) &\longleftrightarrow \frac{\omega}{(s + \alpha)^2 + \omega^2} \\ F(s) &= \frac{\omega}{(s + \alpha)^2 + \omega^2} \end{aligned}$$

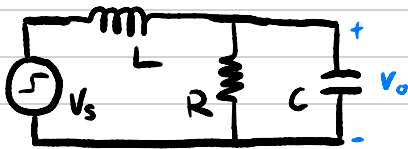
Find the inverse Laplace Transform of

$$F(s) = \frac{(s+2)(s+4)}{s^2 + 3^2}$$

$$\begin{aligned} F(s) &= \frac{s^2 + 6s + 8}{s^2 + 3^2} & \theta = \cos^{-1}\left(\frac{6}{\sqrt{41}}\right) &= 0.778 \\ &= 1 + \frac{6s - 1}{s^2 + 3^2} & \swarrow \cos(\theta) & \quad \nwarrow \sin(\theta) \\ &= 1 + \frac{1}{\sqrt{41}} \frac{6/\sqrt{41} s - 1/\sqrt{41}}{s^2 + 3^2} \\ f(t) &= \delta(t) + \frac{1}{\sqrt{41}} \cos(\omega t + 0.778) \end{aligned}$$

# Laplace Transform

## Problem



Consider the above circuit. With  $R = 1000\Omega$ ,  
 $L = 10\text{ mH}$  &  $C = 100\text{ nF}$ . With  $V_s = 5u(t)\text{ V}$

Find an expression for  $V_o$  in the Laplace domain

5u(t)  
 $\frac{5}{s}$

10mH  
0.01s

1kΩ  
1000

100nF  
 $1 \times 10^{-7} \frac{1}{2}$

$$V_o(s) = \left[ \frac{5}{s} \right] \left[ \frac{1000 \parallel \frac{1}{2} \cdot 10^7}{1000 \parallel \frac{1}{2} \cdot 10^7 + 0.01s} \right]$$
$$= \left[ \frac{5}{s} \right] \left[ \frac{(10^{-3} + 10^{-7}s)^{-1}}{(10^{-3} + 10^{-7}s)^{-1} + 0.01s} \right]$$
$$= \frac{5}{s} \frac{1}{1 + 10^{-5}s + 10^{-9}s^2}$$
$$= \frac{5}{s} \frac{10^9}{s^2 + 10^4s + 10^9}$$

Side note

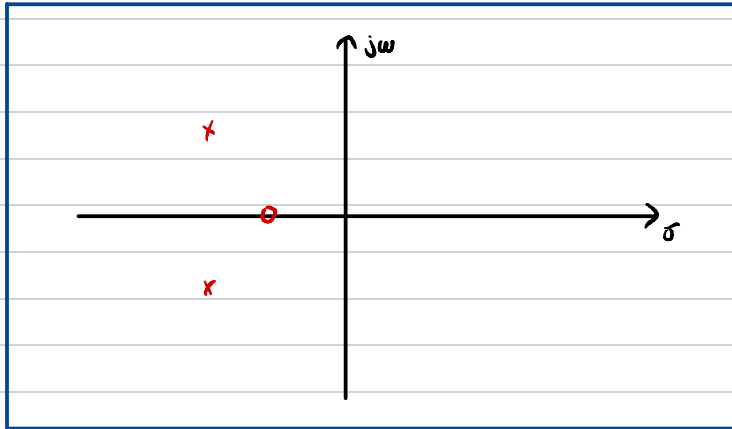
$$\int e^{cx} \sin bx dx$$
$$= \frac{e^{cx}}{c^2 + b^2} [c \cos bx + b \sin bx]$$

# Laplace Transform

## Problem

plot the following equation in the  $s$ -plane & roughly sketch the transfer function:

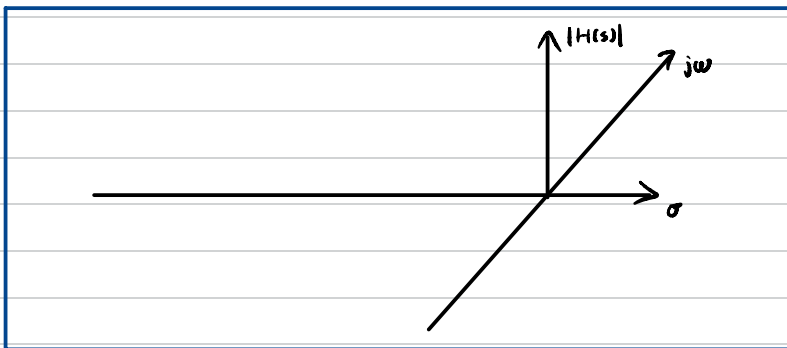
$$\frac{4(s+2)^2}{(s^2+6s+25)}$$



$$\begin{aligned} &\frac{4(s+2)^2}{(s^2+6s+25)} \\ &\frac{4(s+2)}{(s^2+6s+9+16)} \\ &\frac{4(s+2)}{(s+3+4j)(s+3-4j)} \end{aligned}$$

## Drawing Activity!

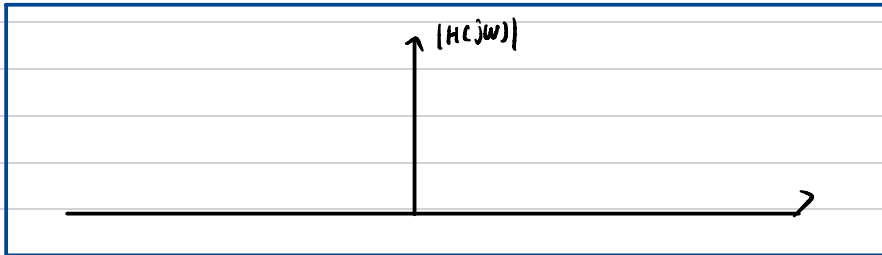
draw a 3 dimensional  $s$  plane!





# Laplace Transform

now roughly sketch the transfer function Amplitude spectrum

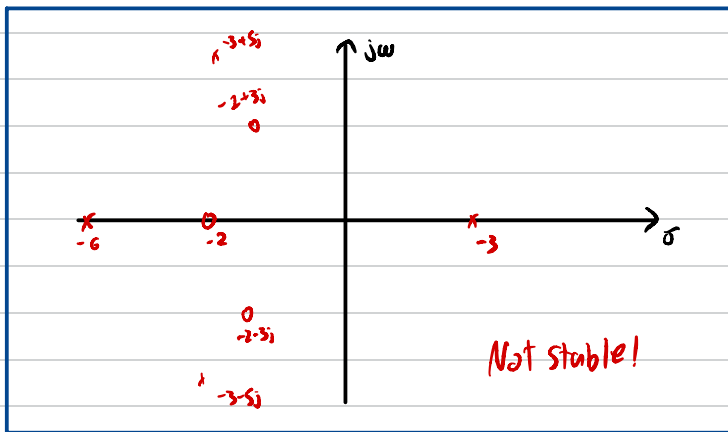


Problem

plot the following transfer function into the  $s$  plane.

Determine whether it is a stable system.

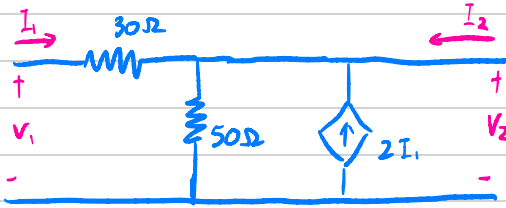
$$\frac{(s+3)^2(s^2+4s+13)}{(s-3)(s^2+3s+34)(s+6)}$$



$$\begin{aligned} & \frac{(s+3)^2(s^2+4s+13)}{(s-3)(s^2+3s+34)(s+6)} \\ &= \frac{(s+3)^2(s+2)^2+9)}{(s-3)(s+3)^2+25)(s+6)} \\ &= \frac{(s+3)^2(s+2+3j)(s+2-3j)}{(s-3)(s+3+5j)(s+3-5j)(s+6)} \end{aligned}$$

# Two Port Network

## Impedance Parameter



$$V = Z I$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\text{let } I_2 = 0$$

$$V_1 = I_1 \times 30 + 50(3I_1)$$

$$= 180 I_1$$

$$z_{11} = 180 \Omega$$

$$V_2 = I_1(50) + 2I_1(50)$$

$$= 150 I_1$$

$$z_{21} = 150 \Omega$$

$$\text{let } I_1 = 0$$

$$V_1 = I_2 \times 50$$

$$z_{12} = 50 \Omega$$

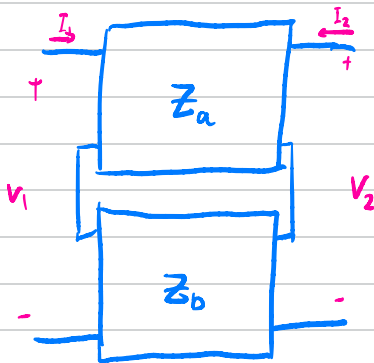
$$V_2 = I_2 \times 50$$

$$z_{22} = 50 \Omega$$

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

## Application



Consider the whole system find  $z$  parameters for system.

$$[z] = [z_a] + [z_b]$$

# Two Port Network

## Admittance Parameter



$$I = YV$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\text{Let } V_2 = 0$$

$$V_1 = 30 I_1$$

$$y_{11} = 30^{-1} S$$

$$I_1 + 2I_1 + I_2 = 0$$

$$3I_1 + I_2 = 0$$

$$3I_1 = -30^{-1} V_1$$

$$y_{21} = -10^{-1}$$

$$\text{Let } V_1 = 0$$

$$V_2 = -I_1 30$$

$$y_{12} = -30^{-1} S$$

$$I_2 + 2I_1 = 18.75^{-1} V_2$$

$$I_2 - 15^{-1} V_2 = 18.75^{-1} V_2$$

$$I_2 = \frac{3}{25} V_2$$

$$y_{22} = \frac{3}{25}$$

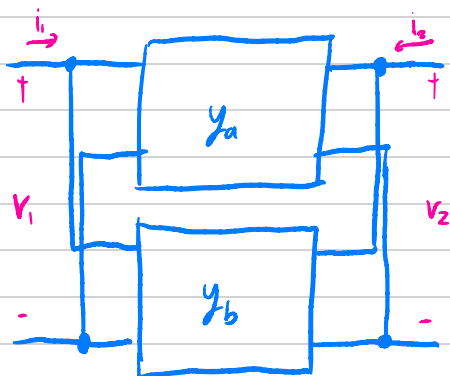
$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$\begin{bmatrix} 180 & 50 \\ 150 & 50 \end{bmatrix} \begin{bmatrix} 30^{-1} & -30^{-1} \\ -10^{-1} & \frac{3}{25} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

## Application

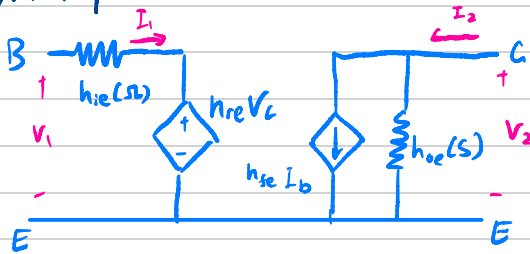


Consider the whole system find  $z$  parameters for system.

$$[y] = [y_a] + [y_b]$$

# Two Port Network

## Hybrid parameters



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

show  $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{ie} & h_{re} \\ h_{fe} & h_{oe} \end{bmatrix}$

Let  $I_2 = 0$

$$V_B = h_{re} V_C$$

$$V_1 = h_{re} V_2 = h_{12} V_2$$

$$h_{12} = h_{re} //$$

$$I_2 = V_2 h_{oe} = h_{22} V_2$$

$$h_{22} = h_{oe} //$$

Let  $V_2 = 0$

$$V_1 = I_1 h_{ie} = h_{11} I_1$$

$$\therefore h_{11} = h_{ie} //$$

$$h_{fe} I_B = I_C$$

$$\therefore I_2 = h_{21} I_1 = h_{fe} I_1$$

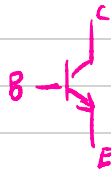
$$h_{21} = h_{fe} //$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

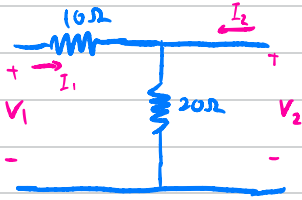
## Application

$h$  parameters tells us a lot of information about a transistor & its behaviour. This is later covered in ELEC 2133.



# Two Port Network

## Transmission Parameters



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Let  $I_2 = 0$

$$V_2 = \frac{20}{10+20} V_1$$

$$V_1 = 1.5 V_2$$

$$A = 1.5 \text{ } \Omega$$

$$V_2 = 20 I_1$$

$$C = 0.05 \text{ S}$$

Let  $V_2 = 0$

$$I_2 = -I_1$$

$$D = 1 \text{ } \Omega$$

$$-I_2 \times 10 = V_1$$

$$V_1 = -10 I_2$$

$$B = 10 \text{ } \Omega$$

$$V_1 = AV_2 - I_2 B$$

$$I_1 = CV_2 - I_2 D$$

$$T = \begin{bmatrix} 1.5 & 10 \\ 0.05 & 1 \end{bmatrix}$$

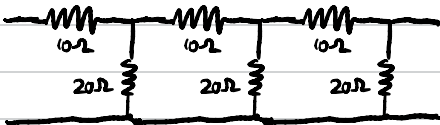
## Application



$$[T] = [T_n][T_o]$$

## problem

find the transmission parameter for the following circuit



$$T = \begin{bmatrix} 1.5 & 10 \\ 0.05 & 1 \end{bmatrix} \begin{bmatrix} 1.5 & 10 \\ 0.05 & 1 \end{bmatrix} \begin{bmatrix} 1.5 & 10 \\ 0.05 & 1 \end{bmatrix} = \begin{bmatrix} 1.5 & 10 \\ 0.05 & 1 \end{bmatrix} \begin{bmatrix} 2.75 & 25 \\ 0.125 & 1.5 \end{bmatrix} = \begin{bmatrix} 5.375 & 52.5 \\ 0.2625 & 2.75 \end{bmatrix}$$